

puting the second pressure derivatives of the cross-coupling moduli, and from the assumption that the second pressure derivatives of the on-diagonal longitudinal moduli, which also enter the calculation, are zero. Although the change in sign follows from the equations used to compute the quantities  $(\rho V^2)''$ , it is important to note that any nonlinear behavior for the on-diagonal longitudinal moduli is not precluded by the present study, since the longitudinal data were very difficult to obtain at higher pressures where curvature might be observed. Because there is no reason to suspect that these coefficients behave linearly under pressure when the shear coefficients do not, it is likely that the second pressure derivatives of the on-diagonal longitudinal moduli are also nonzero. Consequently, the values and the signs of the second pressure derivatives of the cross-coupling moduli depend on the magnitude and the sign of the

on-diagonal longitudinal modes. It is probable, therefore, that, although the determined values of  $(\rho_0 W^2)''$  are excellent, the computed values for  $c_{12}''$ ,  $c_{13}''$ , and  $c_{23}''$  (Table 8) are considerably in error. As a result, a complete description of the nonlinear behavior of the bronzite specimens cannot be given in this study. One important conclusion resulting from the present data, however, is that the elastic properties of possible earth materials can show curvature even at pressures as low as 10 kb. However, owing to a solid-solid phase transition, which should occur at about 135 kb [Ringwood, 1967; Ahrens and Gaffney, 1971], the curvature in the bronzite data should not play a significant role in the earth's interior.

The stated errors for each run in Table 8 are the standard deviations resulting from fitting the data to a quadratic function in pressure. The weighted averages and their associated

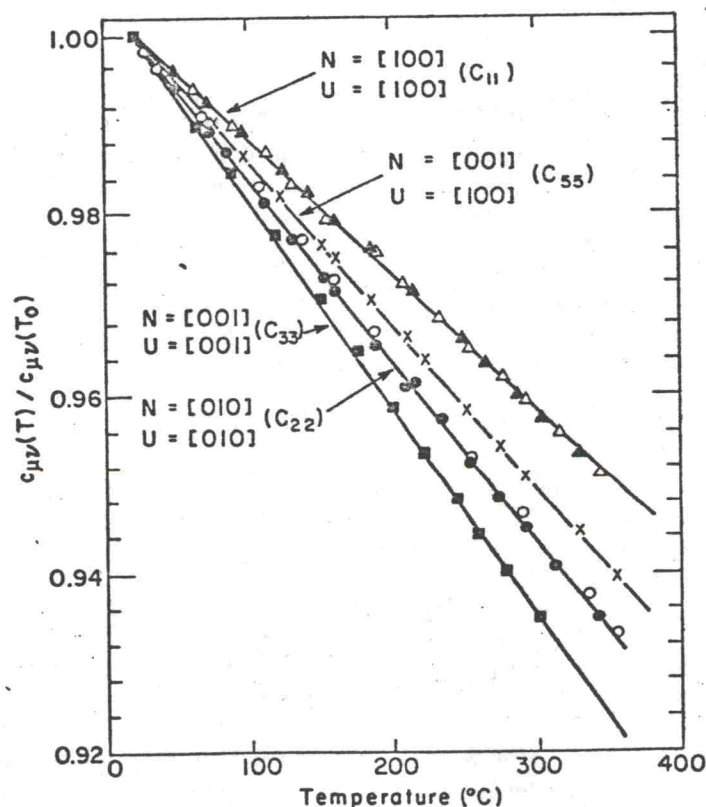


Fig. 4. Experimental data of the on-diagonal elastic constants  $c_{11}$ ,  $c_{22}$ ,  $c_{33}$ , and  $c_{55}$  (referred to their values at  $T_0 = 25^\circ\text{C}$ ) as a function of temperature. Solid triangles, solid squares, open circles, and crosses indicate specimen 1; open triangles, specimen 3; solid circles, specimen 4.

TABLE 9. Isobaric Temperature Derivatives of the On-Diagonal Elastic Constants at 25° to 350°C

Elastic Constant	Specimen	$\vec{N}$	$\vec{U}$	$(\partial c_{\mu\nu}^s / \partial T)_{P=0}$ , kb °C <sup>-1</sup>	Weighted Average, kb °C <sup>-1</sup>
$c_{11}$	1	[100]	[100]	-0.352 ± 0.001	-0.352 ± 0.001
	3	[100]	[100]	-0.353 ± 0.001	
$c_{22}$	1	[010]	[010]	-0.328 ± 0.001	-0.328 ± 0.001
	4	[010]	[010]	-0.329 ± 0.001	
$c_{33}$	1	[001]	[001]	-0.516 ± 0.004	-0.516 ± 0.004
$c_{44}$	1	[001]	[010]	-0.128 ± 0.002	-0.131 ± 0.003
	4	[010]	[001]	-0.122 ± 0.002	
$c_{55}$	1	[001]	[100]	-0.138 ± 0.002	-0.138 ± 0.002
$c_{66}$	3	[100]	[010]	-0.133 ± 0.001	-0.145 ± 0.005
	4	[010]	[100]	-0.150 ± 0.005	

errors were determined by (7) and (8). The difficulty of accurately specifying the second pressure derivatives is reflected in the relatively large scatter in the present data. However, because this difficulty exists, even for synthetic single-crystal specimens (see, for example, *Chang and Barsch* [1971] and *Barsch and Shull* [1971]), and because four different naturally occurring specimens were used, the consistency of the determined second pressure derivatives must be considered as excellent.

As was mentioned above, cross checks on the second pressure derivatives of the cross-coupling moduli by using the quasi-longitudinal modes were not possible. Therefore the consistency of the results was examined by repeating the pressure runs for the quasi-shear modes. The reproducibility and the agreement of these data and those for  $c_{44}$  and  $c_{55}$  obtained by using two different sets of ultrasonic electronic equipment support the nonlinear elastic behavior of the shear moduli beyond any doubt.

*Temperature dependence of second-order elastic constants at 1 atm.* The temperature dependence of the quantities  $\rho V^2$ , which are necessary to determine the temperature derivatives of the elastic constants, has been directly

determined by making the appropriate density and length corrections. Because the on-diagonal elastic moduli are given by  $c_{\mu\mu}^s = \rho V^2$ , the temperature derivatives for  $c_{\mu\mu}^s$  are explicitly determined by fitting a polynomial to the data for  $\rho V^2$  as a function of temperature. The temperature dependence of  $\rho V^2$  for the on-diagonal moduli  $c_{11}$ ,  $c_{22}$ ,  $c_{33}$ , and  $c_{55}$  is plotted in Figure 4. The values have been normalized by dividing by  $c_{\mu\nu}(T_0)$ , with  $T_0 = 25^\circ\text{C}$ .

The values for the temperature derivatives of the on-diagonal elastic constants and their standard deviations as determined by a polynomial fit to these data are presented in Table 9. In all cases the temperature dependence was found to be linear, within experimental limits, up to 350°C. The equations necessary to calculate the temperature derivatives of the cross-coupling coefficients have been given by *Graham* [1969] and *Frisillo* [1972].

The experimental data necessary to compute the temperature dependence of the cross-coupling coefficients have been given by *Graham* [1969]. The values of  $\rho V^2$  for the quasi modes. These data, again normalized by dividing by the initial value  $\rho_0 V_0^2$ , are presented in Figure 5. The computed values for the derivatives of the